Quadratic Equations

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1 Proving the α β formulas

Define the quadratic equation with the variables a, b, c (1)

$$ax^2 + bx + c = 0 \tag{2}$$

Let $\alpha = x_1$ (The first root of the equation) (3)

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{4}$$

Let $\beta = x_2$ (The second root of the equation) (5)

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

Prove the formula $\alpha + \beta = \frac{-b}{a}$ is true (via substitution) (7)

$$\begin{aligned} &\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2b}{2a} \\ &= \frac{-b}{a} \end{aligned} \tag{8}$$

Prove the formula $\alpha\beta = \frac{c}{a}$ is true (via substitution) (9)

$$\begin{aligned} &\frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{(-b + \sqrt{b^2 - 4ac})(-b - \sqrt{b^2 - 4ac})}{4a^2} \\ &= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2} \\ &= \frac{b^2 - (b^2 - 4ac)}{4a^2} \\ &= \frac{4ac}{4a^2} \\ &= \frac{c}{a} \end{aligned} \tag{10}$$